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TEST PARTICLE METHOD IN KINETIC THEORY OF A PLASMA

KYOKO MATSUDA

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TEST PARTICLE METHOD IN KINETIC THEORY OF A PLASMA

Kyoko Matsuda*

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ABSTRACT

A general relationship has been found between many body correlation functions and the conditional probability functions which describe the shield clouds surrounding test particles. This is the generalization of Rostoker's superposition principle. The relationship is useful because the problem of kinetic theory is reduced to determining the conditional probability functions, which involves essentially only the Vlasov equation. This method has been applied to obtain a second order correction to the Lenard-Balescu collison integral.

*NRC – NASA Resident Research Associate.

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CONTENTS

TEST PARTICLE METHOD IN KINETIC THEORY OF A PLASMA

INTRODUCTION

Several years ago, Rostoker[1] found a relationship between the two-particle correlation function $G(X_1\, X_2\, t)$ and a conditional probability function $P(X' \mid Xt)$ which characterizes a shield cloud of field particles of coordinates $X = (\mathbf{x}, \mathbf{v})$ surrounding a test particle of coordinates $X' = (\mathbf{x}', \mathbf{v}')$. It is

$$G(X_1\, X_2\, t) = f(X_1\, t)P(X_1 \mid X_2\, t) + f(X_2\, t)P(X_2 \mid X_1\, t)$$

$$+\, n \int dX' \; f(X'\, t)P(X' \mid X_1\, t)P(X' \mid X_2\, t)\,. \qquad (1)$$

This relation has been established without solving explicitly for anything and has none of the usual restrictions such as spatial homogeneity, adiabatic time behavior etc. usually necessary for obtaining explicit solutions. It is useful because the problem of kinetic theory is reduced to determining P, which involves only the Vlasov equation.

A similar relationship has been obtained by Rostoker and the author[2] to higher order in the plasma parameter using the procedure of ensemble averages of the singular distribution function for a system of initially uncorrelated particles. The first two relations are, to second order in the plasma parameter

$$\hat{G}(X_1\ X_2\ t) = n \int dX_1'\, F(X_1')\, \tilde{P}(X_1' \mid X_1\ t)\, \tilde{P}(X_1' \mid X_2\ t)$$

$$+ \frac{n^2}{2} \int dX_1'\, dX_2'\, F(X_1')\, F(X_2')\, \tilde{Q}(X_1'\ X_2' \mid X_1\ t)\, \tilde{Q}(X_1'\ X_2' \mid X_2\ t) \qquad (2)$$

$$\hat{T}(123) = n \int d1'\, F(1')\, \tilde{P}(1' \mid 1)\, \tilde{P}(1' \mid 2)\, \tilde{P}(1' \mid 3)$$

$$+ \sum_P n^2 \int d1'\, d2'\, F(1')\, F(2')\, \tilde{P}(1' \mid 1)\, \tilde{P}(2' \mid 2)\, \tilde{Q}(1'\ 2' \mid 3) \qquad (3)$$

where

$$\hat{G}(12) = \Delta(1,2)\ f(1) + G(12)$$

$$\hat{T}(123) = \Delta(1,2)\ \Delta(2,3)\ f(1) + \Delta(1,2)\ G(23) + \Delta(2,3)\ G(31)$$

$$+ \Delta(3,\bar{1})\ G(\bar{1}2) + T(123)$$

$$\Delta(1,2) = \frac{1}{n}\,\delta\,(\mathbf{x}_1 - \mathbf{x}_2)\,\delta\,(\mathbf{v}_1 - \mathbf{v}_2)$$

and $\hat{G}(X_1\ X_2\ t)$ is written as $\hat{G}(12)$. G and T are the usual two-particle and three-particle correlation functions, and F is the one-particle function at the initial time. $\tilde{P}(1' \mid 1)$ is the probability of finding a particle at $X_1 = (\mathbf{x}_1, \mathbf{v}_1)$

at time t given a particle at $X_1' = (x_1', v_1')$ at the initial time. $\tilde{P}(1' \mid 1) + \tilde{P}(2' \mid 1) + \tilde{Q}(1' \, 2' \mid 1)$ is the probability of finding a particle at $(X_1 \, t)$, given a pair of particles at $(X_1' \, X_2' \, t = 0)$. The equations for $\tilde{P}$ and $\tilde{Q}$ are obtained by carrying out averages on the Klimontovitch equation.

From the definition and also Equations (2) and (3), it is expected that $\tilde{P}(1' \mid 1)$ must have singularities. For instance, consider a uniform field-free system. Solving the equation of $\tilde{P}$, we obtain

$$\tilde{P}(1' \mid 1) = \frac{1}{n} \delta(v_1 - v_1') \, \delta(x_1 - x_1' - v_1' \, t) + \text{nonsingular terms} . \quad (4)$$

The first term is interpreted as the probability of finding the test particle while the second represents a shielding particle. However this singular solution may be criticized because the expansion itself breaks down in the vicinity of test particles. In fact, using this solution Equations (2) and (3) do not give the proper singularities of $\hat{G}$ and $\hat{T}$. A further criticism of the singular function of Equation (4) is given in Appendix.

Since the main object of Reference 2 is to make the relationship clear between kinetic theory and weak turbulence theory, the inconsistency of the solution $\tilde{P}$ does not affect the discussion. A correction should be made to the discussion concerned with discrete particle effects. Of course, by the termination of the hierarchy, the approach in Reference 1 also breaks down in the vicinity of the test particle. In both approaches, we are not free from the problem of cut-off at short distances. However this is not our present problem. The approach in Reference 1 is selfconsistent. The difficulty in Reference 2 is that due to the inconsistency, the restriction to a system of initially uncorrelated particles is not able to be removed. Besides the physical pictures of $\tilde{P}(1' \mid 1)$, $\tilde{Q}(1' \, 2' \mid 1)$, $\cdots$ are less clear than that of $P(1' \mid 1)$.

In this paper, the direct generalization of Reference 1 to higher order in the plasma parameter is presented. The general relation as well as Equation (1) has none of the usual restrictions, which we have stated below Equation (1). Rostoker's superposition principle, that is, Equation (1) has been often misunderstood at this point[3]. Through this relation, the Bogoliubov-Born-Green-Kirkwood-Yvon hierarchy is reduced to the hierarchy of the test particle functions which involves essentially only the Vlasov equation.

As an application of this method, a second order correction to the Lenard-Balescu collision term has been calculated. The correction is significant for a system with a one dimensional velocity distribution such as a plasma with an extremely strong magnetic field, for which the Lenard-Balescu term vanishes.

KINETIC THEORY

We consider a gas of charged particles interacting only through Coulomb forces. The system may be described by the B - B - G - K - Y hierachy. If we introduce an additional charged particle into the system, the field particles form a cloud surrounding the test particle. To first order in the plasma parameter the cloud is characterized by the perturbation $P(X_1' \mid X_1 \, t)$ of the one-particle distribution function $f(X_1\ t)$ which is governed by[1]

$$\left\{\frac{\partial}{\partial t} + O'(X_1'\ t) + O(X_1\ t)\right\} P(X_1' \mid X_1\ t) = \frac{e^2}{m}\frac{\partial f(X_1\ t)}{\partial \mathbf{v}_1} \cdot \frac{\partial}{\partial \mathbf{x}_1} \frac{1}{|\mathbf{x}_1 - \mathbf{x}_1'|}, \quad (5)$$

where

$$O'(X_1'\ t) = \mathbf{v}_1' \cdot \frac{\partial}{\partial \mathbf{x}_1'} - \frac{e}{m} \mathbf{F}_M (X_1'\ t) \cdot \frac{\partial}{\partial \mathbf{v}_1'}$$

and

$$O(X_1\ t) = \mathbf{v}_1 \cdot \frac{\partial}{\partial \mathbf{x}_1} - \frac{e}{m} \mathbf{F}_M (X_1\ t) \cdot \frac{\partial}{\partial \mathbf{v}_1}$$

$$- \frac{ne^2}{m} \frac{\partial f\ (X_1\ t)}{\partial \mathbf{v}_1} \frac{\partial}{\partial \mathbf{x}_1} \cdot \int dX_2 \frac{1}{|\mathbf{x}_1 - \mathbf{x}_2|} \left\{\cdots\right\} .$$

$\mathbf{F}_M(Xt)$ is the macroscopic force, that is, the external electromagnetic force plus the macroscopic Coulomb force

$$ne \frac{\partial}{\partial \mathbf{x}} \int dX_2 \frac{f(X_2\ t)}{|\mathbf{x} - \mathbf{x}_2|} .$$

Infinite mass randomly distributed ions are assumed since the generalization including ion distributions is trivial. In Reference 1, the relation expressed by Equation (1) has been proved between the solution of Equation (5) and the two-particle correlation function.

In second order theory, we are concerned with a new quantity $W(X_1' \mid X_1\ X_2\ t)$ which is the perturbation of the two-particle correlation function $G(X_1\ X_2\ t)$

due the the test particle. To save space, we introduce singular functions

$$\hat{P}(1' \mid 1) = P(1' \mid 1) + \Delta(1', 1) ,$$

$$\hat{W}(1' \mid 12) = W(1' \mid 12) + \Delta(1, 2)\, P(1' \mid 1) .$$

$P(1' \mid 1)$ and $W(1' \mid 12)$ obey the equations

$$\left\{\frac{\partial}{\partial t} + O'(1') + O(1)\right\} \hat{P}(1' \mid 1)$$

$$= \frac{ne^2}{m} \frac{\partial \hat{P}(1' \mid 1)}{\partial \mathbf{v}_1} \cdot \int d2 \frac{\partial}{\partial \mathbf{x}_1} \frac{1}{|\mathbf{x}_1 - \mathbf{x}_2|} \hat{P}(1' \mid 2)$$

$$+ \frac{ne^2}{m} \frac{\partial \hat{P}(1' \mid 1)}{\partial \mathbf{v}_1'} \cdot \int d2 \frac{\partial}{\partial \mathbf{x}_1'} \frac{1}{|\mathbf{x}_1' - \mathbf{x}_2|} \hat{P}(1' \mid 2)$$

$$+ \frac{ne^2}{m} \int d2 \frac{\partial}{\partial \mathbf{x}_1} \frac{1}{|\mathbf{x}_1 - \mathbf{x}_2|} \cdot \frac{\partial}{\partial \mathbf{v}_1} \hat{W}(1' \mid 12)$$

$$+ \frac{ne^2}{m} \int d2 \frac{\partial}{\partial \mathbf{x}_1'} \frac{1}{|\mathbf{x}_1' - \mathbf{x}_2|} \cdot \frac{\partial}{\partial \mathbf{v}_1'} \hat{W}'(1' \mid 12) \qquad (6)$$

$$\left\{\frac{\partial}{\partial t} + O'(1') + O(1) + O(2)\right\} \hat{W}(1' \mid 12)$$

$$= \frac{ne^2}{m} \frac{\partial \hat{P}(1' \mid 1)}{\partial \mathbf{v}_1} \cdot \int d3 \frac{\partial}{\partial \mathbf{x}_1} \frac{1}{|\mathbf{x}_1 - \mathbf{x}_3|} \hat{G}(23)$$

$$+ \frac{ne^2}{m} \frac{\partial \hat{P}(1' \mid 1)}{\partial \mathbf{v}_1'} \cdot \int d3 \frac{\partial}{\partial \mathbf{x}_1'} \frac{1}{|\mathbf{x}_1' - \mathbf{x}_3|} \hat{G}(23)$$

$$+ \frac{ne^2}{m} \frac{\partial \hat{P}(1' \mid 2)}{\partial \mathbf{v}_2} \cdot \int d3 \frac{\partial}{\partial \mathbf{x}_2} \frac{1}{|\mathbf{x}_2 - \mathbf{x}_3|} \hat{G}(13)$$

$$+ \frac{ne^2}{m} \frac{\partial \hat{P}(1' \mid 2)}{\partial \mathbf{v}_1'} \cdot \int d3 \frac{\partial}{\partial \mathbf{x}_1'} \frac{1}{|\mathbf{x}_1' - \mathbf{x}|} \hat{G}(13)$$

$$+ \frac{ne^2}{m} \frac{\partial \hat{G}(12)}{\partial \mathbf{v}_1} \cdot \int d3 \frac{\partial}{\partial \mathbf{x}_1} \frac{1}{|\mathbf{x}_1 - \mathbf{x}_3|} \hat{P}(1' \mid 3)$$

$$+ \frac{ne^2}{m} \frac{\partial \hat{G}(12)}{\partial \mathbf{v}_2} \cdot \int d3 \frac{\partial}{\partial \mathbf{x}_2} \frac{1}{|\mathbf{x}_2 - \mathbf{x}_3|} \hat{P}(1' \mid 3) \,. \qquad (7)$$

In the right hand side of Equation (6), the first two terms are effects of Coulomb forces due to the dressed test particle; the third and fourth term are perturbations of the collision integral due to the test particle. The function W' is not obvious at this stage.

The solution of Equation (7) may be given by making use of Equation (1). Using the singular functions Equation (1) is written as

$$\hat{G}(12) = n \int d2' \; f(2') \hat{P}(2' \mid 1) \hat{P}(2' \mid 2) \; . \tag{8}$$

The test particle at X_1' produces the perturbation

$$\hat{W}(1' \mid 12) = n \int d2' \; P(1' \mid 2') \; \hat{P}(2' \mid 1) \; \hat{P}(2' \mid 2)$$

$$+ \; \int d2' \; f(2') \; Q(1' \; 2' \mid 1) \; \hat{P}(2' \mid 2)$$

$$+ \; n \int d2' \; f(2') \; \hat{P}(2' \mid 1) \; Q(1' \; 2' \mid 2) \; . \tag{9}$$

$Q(1'2' \mid 1)$ describes the cloud surrounding a pair of particles, in other words, the perturbation of $f(X_1 \; t)$ due to a pair of test particles at X_1' and X_2', and obeys the equation

$$\left\{\frac{\partial}{\partial t} + O'(1') + O'(2') + O(1)\right\} Q(1' \; 2' \mid 1)$$

$$= \frac{ne^2}{m} \frac{\partial \hat{P}(1' \mid 1)}{\partial \mathbf{v}_1} \cdot \int d2 \frac{\partial}{\partial \mathbf{x}_1} \frac{1}{|\mathbf{x}_1 - \mathbf{x}_2|} \hat{P}(2' \mid 2)$$

$$+ \frac{ne^2}{m} \frac{\partial \hat{P}(1' \mid 1)}{\partial \mathbf{v}_1'} \cdot \int d2 \frac{\partial}{\partial \mathbf{x}_1'} \frac{1}{|\mathbf{x}_1' - \mathbf{x}_2|} \hat{P}(2' \mid 2)$$

$$+ \frac{ne^2}{m} \frac{\partial \hat{P}(2' \mid 1')}{\partial \mathbf{v}_1} \cdot \int d2 \frac{\partial}{\partial \mathbf{x}_1} \frac{1}{|\mathbf{x}_1 - \mathbf{x}_2|} \hat{P}(1' \mid 2)$$

$$+ \frac{ne^2}{m} \frac{\partial \hat{P}(2' \mid 1)}{\partial \mathbf{v}_2'} \cdot \int d2 \frac{\partial}{\partial \mathbf{x}_2'} \frac{1}{|\mathbf{x}_2' - \mathbf{x}_2|} \hat{P}(1' \mid 2) \ . \qquad (10)$$

This equation is much easier to solve than Equation (7), and the quantity Q has much simpler meaning than the quantity W does.

It is straightforward although tedious to show that relationships exist between the first two correlation functions of the hierarchy and the functions P and Q, provided that the quantity W' is given by

$$\hat{W}'(1' \mid 12) = \hat{W}(1' \mid 12) - n \int d2' \ f(2') \ \hat{P}(2' \mid 1) \ Q(1' \ 2' \mid 2) \ ; \qquad (11)$$

that is, $\hat{W}'(1' \mid 12)$ does not contain the term of $\hat{W}(1' \mid 12)$ in which the test particle interacts only with the shielding particle. The relationships are

$$\hat{G}(12) = n \int d1' \ f(1') \ \hat{P}(1' \mid 1) \ \hat{P}(1' \mid 2)$$

$$+ \ n^2 \int d1' d2' \ P(1' \mid 2') \ P(2' \mid 1') \ \hat{P}(1' \mid 1) \ \hat{P}(2' \mid 2)$$

$$+\frac{n^2}{2}\int d1'\, d2'\, f(1')\, f(2')\, Q(1'\, 2' \mid 1)\, Q(1'\, 2' \mid 2)$$

$$+\sum_P n^2 \int d1'\, d2'\, f(1')\, P(1' \mid 2')\, \hat{P}(2' \mid 1)\, Q(1'\, 2' \mid 2) \tag{12}$$

$$\hat{T}(123) = n\int d1'\, f(1')\, \hat{P}(1' \mid 1)\, \hat{P}(1' \mid 2)\, \hat{P}(1' \mid 3)$$

$$+\sum_P n^2 \int d1'\, d2'\, f(1')\, P(1' \mid 2')\, \hat{P}(1' \mid 1)\, \hat{P}(2' \mid 2)\, \hat{P}(2' \mid 3)$$

$$+\sum_P n^2 \int d1'\, d2'\, f(1')\, f(2')\, \hat{P}(1' \mid 1)\, \hat{P}(2' \mid 2)\, Q(1'\, 2' \mid 3) \tag{13}$$

where

$$\sum_P$$

means the sum over all cyclic permutations of particles involved. Each term in Equations (12) and (13) is shown schematically in Figures I and II in the same order.

The rule governing the general relationship between any higher order correlation function and the test particle function $P(1' \mid 1)$, $Q(1'\, 2' \mid 1)$, $R(1'\, 2'\, 3' \mid 1)$ etc. can be obtained from these diagrams. A dot indicates a test particle and a circle indicates a shielding cloud. Each dot counts as order $1/\epsilon$, each circle surrounding one center counts as order ϵ, each circle surrounding two centers counts as order ϵ^2, etc., where ϵ is the plasma parameter. In each diagram, every center is surrounded by the same number of circles and the total order of a diagram is given by the product of orders for circles and dots in the diagram. It should be noted that the interaction between additional particles which will be averaged always comes in as a nonsingular function.

If we rewrite the relationship in terms of nonsingular functions, for the two-particle correlation function

$$
\begin{aligned}
G(12) = {} & f(1)\, P(1 \mid 2) + f(2)\, P(2 \mid 1) + n \int d1'\; f(1')\, P(1' \mid 1)\, P(1' \mid 2) \\
& + P(1 \mid 2)\, P(2 \mid 1) + \sum_P n \int d1'\; P(1 \mid 1')\, P(1' \mid 1)\, P(1' \mid 2) \\
& + n^2 \int d1'\, d2'\, P(1' \mid 2')\, P(2' \mid 1')\, P(1' \mid 1)\, P(2' \mid 2) \\
& + \sum_P n \int d1'\; f(1')\, P(1' \mid 1)\, Q(1'\, 1 \mid 2) \\
& + \frac{n^2}{2} \int d1'\, d2'\; f(1')\; f(2')\; Q(1'\, 2' \mid 1)\; Q(1'\, 2' \mid 2) \\
& + \sum_P n^2 \int d1'\, d2'\; f(1')\, P(1' \mid 2')\, P(2' \mid 1)\, Q(1'\, 2' \mid 2) \; .
\end{aligned}
\qquad (14)
$$

This is schematically expressed in Figure III and the three particle correlation is shown in Figure IV. The rule governing these diagrams is the same as before. The differences are as follows. An open dot indicates one of particles involved which itself acts as a test particle and has order ϵ^0. There is no restriction on the number of circles surrounding this kind of dot.

The relationship always exists provided it is satisfied at the initial time. Now kinetic theory is reduced to the determination of $P(1' \mid 1)$, $Q(1'\ 2' \mid 1)$, $R(1'\ 2'\ 3' \mid 1)$, etc. The equations for these quantities are essentially the Vlasov equation.

Rostoker has shown that the first order superposition principle gives the known two-particle correlation function for an equilibrium system. In the second order theory, we also obtain the equilibrium two- and three-particle correlation functions which have already been obtained by other methods[4]. The entire procedure is considerably more complicated than that in the first order theory and the details will be omitted in this paper. However it may be worthy to notice that we obtain some integrals which are hardly calculated by a straight-forward way. They are, for instance

$$\int_{-\infty}^{\infty} du \frac{F(u)}{|\epsilon(k,\ -iku)|^2} = \frac{1}{\epsilon(k,\ 0)},$$

$$\frac{1}{\epsilon(k_1, 0)} \int_{-\infty}^{\infty} d\tau \frac{P}{\tau} \operatorname{Im} \left\{ \frac{1}{\epsilon(k_2,\ -i\tau)\ \epsilon(k_3,\ -i\tau)} \right\}$$

$$+\frac{1}{\epsilon(k_2,0)}\int_{-\infty}^{\infty} d\tau \frac{P}{\tau}\,\mathrm{Im}\left\{\frac{1}{\epsilon(k_3,\ -i\tau)\ \epsilon(k_1,\ -i\tau)}\right\}$$

$$+\frac{1}{\epsilon(k_3,0)}\int_{-\infty}^{\infty} d\tau \frac{P}{\tau}\,\mathrm{Im}\left\{\frac{1}{\epsilon(k_1,\ -i\tau)\ \epsilon(k_2,\ -i\tau)}\right\}.$$

$$= -\pi\,\frac{\dfrac{\kappa^4}{k_1^2 k_2^2}+\dfrac{\kappa^4}{k_2^2 k_3^2}+\dfrac{\kappa^4}{k_3^2 k_1^2}+\dfrac{2\kappa^6}{k_1^2 k_2^2 k_3^2}}{\epsilon(k_1,0)\ \epsilon(k_2,0)\ \epsilon(k_3,0)}$$

for finite positive k_1, k_2, k_3, where $F(u)$ is the one dimensional maxwell function

$$F(u) = \frac{1}{\sqrt{2\pi}\ v_{th}}\exp\left(-\frac{u^2}{2v_{th}^2}\right),$$

$\kappa = \omega_p/v_{th}$, and

$$\epsilon(k,\ -i\tau) = 1+\frac{\kappa^2}{k^2}\int\frac{du\ u\,F(u)}{u-\frac{\tau}{k}-i\delta}.$$

They are obtained by changing order of integrations in

$$\int dv_1\,dv_2\,G_k\,(v_1,\ v_2)$$

or

$$\int dv_1\,dv_2\,dv_3\,T_{k_1 k_2}(v_1,\ v_2,\ v_3)$$

where G_k and $T_{k_1 k_2}$ are the Fourier transform of correlation functions and expressed in terms of the test particle functions.

COLLISION INTEGRAL

In this section, we calculate a second order correction to the Lenard-Balescu collision term. To this end adiabatic time behaviour and homogeneity of the system are assumed.

The basic technique used here is the Fourier-Laplace transformation. From Equation (5), the asymptotic solution $P_k(1' \mid 1)$ is easily obtained to first order in the plasma parameter. Incidentally making use of Equation (1), this solution gives the Lenard-Balescu term. When we know the right hand side of Equation (10) and note that

$$\int dv_2 \hat{P}_k(1' \mid 2) = \frac{1}{n} \frac{1}{\epsilon(k, -ik \cdot v_1')} \tag{15}$$

where

$$\epsilon(k, -ik \cdot v_1') = 1 - \frac{\omega_P^2}{k^2} \int \frac{dv \; ik \cdot \frac{\partial}{\partial v} f}{ik \cdot (v - v_1') + \delta}$$

and damping terms are neglected, we may obtain $Q_{k_1 k_2}(1' \, 2' \mid 1)$. The right hand side of Equation (6) is now known to obtain the second order solution for $P(1' \mid 1)$. We note that we may use only time independent terms of the asymptotic solutions for the right hand sides of Equation (6) and (10), since all others are damped after the integrations over velocities of shielding particles.

We thus obtain

$$\int G_k^{(2)}(12)\, dv_2 = \int dv_2 \frac{f(2)\, P_{-k}^{(2)}(2 \mid 1)}{\epsilon(k, -ik \cdot v_2)}$$

$$+ n \int dv_1'\, f(1')\, \hat{P}_{-k}^{(1)}(1' \mid 1) \int dv_2\, P_k^{(2)}(1' \mid 2)$$

$$+ \int \frac{dk'}{(2\pi)^3} \left[n \int dv_1' \hat{P}_{-k}^{(1)}(1' \mid 1) \int dv_2 \frac{P_{k+k'}^{(1)}(1' \mid 2)\, P_{k'}^{(1)}(2 \mid 1')}{\epsilon(k, -ik \cdot v_2)} \right.$$

$$+ n \int dv_1'\, f(1') \int dv_2 \frac{P_{k+k'}^{(1)}(1' \mid 2')\, Q_{-k-k', k'}(1' 2' \mid 1)}{\epsilon(k, -ik \cdot v_2)}$$

$$+ n^2 \int dv_1'\, dv_2'\, f(1')\, P_{-k-k'}^{(1)}(1' \mid 2')\, \hat{P}_{-k}^{(1)}(2' \mid 1) \int dv_2\, Q_{k+k', -k'}(1' 2' \mid 2)$$

$$\left. + \frac{n^2}{2} \int dv_1'\, dv_2'\, f(1')\, f(2')\, Q_{-k-k', k'}(1' 2' \mid 1) \int dv_2\, Q_{k+k', -k'}(1' 2' \mid 2) \right] \tag{16}$$

where $P_k^{(1)}(1' \mid 1)$, $P_k^{(2)}(1' \mid 1)$ and $Q_{k_1, k_2}(1' 2' \mid 1)$ are the time independent parts of the corresponding asymptotic solutions

$$P_k^{(1)}(1' \mid 1) = \frac{\omega_p^2}{n} \frac{1}{\epsilon(k, -ik \cdot v')} \frac{\frac{ik}{k^2} \cdot \frac{\partial f}{\partial v}}{ik \cdot (v_1 - v_1') + \delta} \tag{17}$$

$$Q_{\mathbf{k}_1,\mathbf{k}_2}(1'\,2' \mid 1) = \frac{1}{n} L(\mathbf{k}_1+\mathbf{k}_2, \mathbf{v}_1 ; \mathbf{k}_1\cdot\mathbf{v}_1' + \mathbf{k}_\cdot\cdot\mathbf{v}_2') \Bigg[\frac{1}{\epsilon(k_1, -i\mathbf{k}_1\cdot\mathbf{v}_1')} \frac{i\mathbf{k}_1}{k_1^2} \cdot \frac{\partial}{\partial \mathbf{v}_1} P_{\mathbf{k}_2}^{(1)}(2' \mid 1)$$

$$+ \frac{1}{\epsilon(k_2, -i\mathbf{k}_2\cdot\mathbf{v}_2')} \frac{i\mathbf{k}_2}{k_2^2} \cdot \frac{\partial}{\partial \mathbf{v}_1} P_{\mathbf{k}_1}^{(1)}(1' \mid 1)$$

$$+ \frac{1}{\epsilon(k_2, -i\mathbf{k}_2\cdot\mathbf{v}_2)} \frac{i\mathbf{k}_2}{k_2^2} \cdot \frac{\partial}{\partial \mathbf{v}_1'} P_{\mathbf{k}_1+\mathbf{k}_2}^{(1)}(1' \mid 1)$$

$$+ \frac{1}{\epsilon(k_1, -i\mathbf{k}_1\cdot\mathbf{v}_1')} \frac{i\mathbf{k}_1}{k_1^2} \cdot \frac{\partial}{\partial \mathbf{v}_2'} P_{\mathbf{k}_1+\mathbf{k}_2}^{(1)}(2' \mid 1) \Bigg] \qquad (18)$$

$$P_{\mathbf{k}}^{(2)}(1' \mid 1) = \frac{1}{n} L(\mathbf{k}, \mathbf{v}_1 : \mathbf{k}\cdot\mathbf{v}_1') \int \frac{d\mathbf{k}'}{(2\pi)^3} \frac{i\mathbf{k}'}{k'^2}$$

$$\cdot \Bigg[\frac{1}{\epsilon(k', -i\mathbf{k}'\cdot\mathbf{v}_1')} \frac{\partial}{\partial \mathbf{v}_1} P_{\mathbf{k}-\mathbf{k}'}(1' \mid 1) + \frac{1}{\epsilon(k', -i\mathbf{k}'\cdot\mathbf{v}_1)} \frac{\partial}{\partial \mathbf{v}_1'} P_{\mathbf{k}+\mathbf{k}'}(1' \mid 1)$$

$$+ \frac{\partial}{\partial \mathbf{v}_1} \left\{ P_{\mathbf{k}}(1' \mid 1) \left(\frac{1}{\epsilon(k', -i\mathbf{k}'\cdot\mathbf{v}_1)} - 1 \right) \right\}$$

$$+ \frac{\partial}{\partial \mathbf{v}_1'} \{ P_{\mathbf{k}}(1' \mid 1) \} \left(\frac{1}{\epsilon(k', -i\mathbf{k}'\cdot\mathbf{v}_1')} - 1 \right)$$

$$+ \frac{\partial}{\partial \mathbf{v}_1} \Bigg\{ n \int d\mathbf{v}_2 \frac{P_{\mathbf{k}}^{(1)}(1' \mid 2)\, P_{\mathbf{k}'}^{(1)}(2 \mid 1)}{\epsilon(k', -i\mathbf{k}'\cdot\mathbf{v}_2)} + n \int d\mathbf{v}_2 \frac{f(2)\, Q_{\mathbf{k},-\mathbf{k}'}(1'\,2 \mid 1)}{\epsilon(k', -i\mathbf{k}'\cdot\mathbf{v}_2)}$$

$$+ n^2 \int d\mathbf{v}_2'\, f(2')\, \hat{P}_{\mathbf{k}-\mathbf{k}'}^{(1)}(2' \mid 1) \int d\mathbf{v}_2\, Q_{\mathbf{k},\mathbf{k}'-\mathbf{k}}(1'\,2' \mid 2) \Bigg\}$$

$$+\frac{\partial}{\partial \mathbf{v}_1'}\left\{n\int d\mathbf{v}_2 \frac{P^{(1)}_{k+k'}(1' \mid 2)\, P^{(1)}_{k}(2 \mid 1)}{\epsilon(k', -ik'\cdot \mathbf{v}_2)} + n\int d\mathbf{v}_2 \frac{f(2)\, Q_{k+k', -k'}(1'\, 2 \mid 1)}{\epsilon(k', -ik'\cdot \mathbf{v}_2)}\right\}\Bigg] . \tag{19}$$

The operator L is defined as

$L(\mathbf{k}, \mathbf{v}_1 : \mathbf{k}\cdot\mathbf{v}_1')\, \xi(\mathbf{v}_1)$

$$= \frac{\omega_p^2}{i\mathbf{k}\cdot(\mathbf{v}_1 - \mathbf{v}_1') + \delta}\left[\xi(\mathbf{v}_1) + \frac{\omega_p^2 \frac{i\mathbf{k}}{k^2}\cdot\frac{\partial f}{\partial \mathbf{v}_1}}{\epsilon(\mathbf{k}, -i\mathbf{k}\cdot\mathbf{v}_1')}\int \frac{d\mathbf{v}\, \xi(\mathbf{v})}{i\mathbf{k}\cdot(\mathbf{v}-\mathbf{v}_1') + \delta}\right] . \tag{20}$$

The kinetic equation is written as

$$\frac{\partial f}{\partial t} = -\frac{\partial}{\partial \mathbf{v}_1}\cdot\left(\mathbf{J}^{(1)}(\mathbf{v}_1) + \mathbf{J}^{(2)}(\mathbf{v}_1)\right) \tag{21}$$

where $\mathbf{J}^{(1)}$ is the Lenard-Balescu collision integral and

$$\mathbf{J}^{(2)}(\mathbf{v}_1) = \omega_p^2\int \frac{d\mathbf{k}}{(2\pi)^3}\frac{\mathbf{k}}{k^2}\,\mathrm{Im.}\left\{\int d\mathbf{v}_2\, G_k^{(2)}(12)\right\} .$$

The imaginary part of Equation (16) can not be expressed in a simpler way unless the one-particle function f is specified. In a system of one dimensional velocity distribution where

$$\mathrm{Im.}\int d\mathbf{v}_2\, G_k^{(1)}(\mathbf{v}_1\, \mathbf{v}_2)$$

vanishes, we may observe that this second order correction does not vanish in general. For this conclusion, it is enough to check that the term with a particular power of functions $F(u_1)$, $\partial F/\partial u_1$, $\partial^2 F/\partial u_1^2$, $\cdots$ does not vanish for a given k where $F(u_1)$ is the one dimensional velocity distribution function. For instance, the terms containing $\partial^4 F/\partial u_1^4$, which is the highest derivative involved here, result from the first, second and last term of Equation (16), and have nonvanishing coefficients.

DISCUSSION

The superposition principle reveals the structure of many body correlation functions.

An interesting application of the higher order theory is a systematic study of mode coupling for an unstable plasma. Recently Price[5] and Harris[6] have shown that for the time behavior of the distribution function f, many other effects are involved than those considered by quasi-linear theory. Price's work is based on Dupree's method and the long time evolution is treated as a succession of initial value problems. The limitation of his work is its restriction to a homogeneous plasma and the neglect of three particle effects. The first order test particle method gives the same result as Price's for a homogeneous plasma. It is expected that a systematic study of three particle effects yields a much different result than that given by mode coupling in quasi-linear theory.

Since the higher order equations are still complicated, some simplification is desirable to apply them to actual problems. To this object, the characteristic of each term will be studied in a future paper.

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APPENDIX

We show another example in which the singular distribution of Equation (4) fails. Using a singular distribution function

$$\hat{f}(X,t) = \frac{1}{n}\sum_i \delta\left(X - X_i(t)\right) ,$$

we have the well known relations

$$\int d\{X_i\} D_N\left(\{X_i\}, t\right) \hat{f}\left(X, \{X_i\}\right) = f(X,t) \tag{A-1}$$

$$\int d\{X_i\} D_N\left(\{X_i\}, t\right) \hat{f}\left(X_1, \{X_i\}\right) f\left(X_2, \{X_i\}\right) = \Delta(1,2)\ f(X_1, t) + f_2(X_1 X_2, t) \tag{A-2}$$

etc., D_N is a solution of the Liouville equation. By Liouville's theorem the left hand sides are equivalent to

$$\int d\{X_{i0}\} D_N\left(\{X_{i0}\}, t=0\right) f\left(X, \left\{X_i\left(\{X_{i0}\}, t\right)\right\}\right) \tag{A-1'}$$

$$\int d\{X_{i0}\} D_N\left(\{X_{i0}\}, 0\right) \hat{f}\left(X_1, \left\{X_i\left(\{X_{i0}\}, t\right)\right\}\right) \hat{f}\left(X_2, \left\{X_i\left(\{X_{i0}\}, t\right)\right\}\right) \tag{A-2'}$$

etc., where X_{i0} is the initial position of the i^{th} particle in the phase space. For convenience, consider a uniform field-free system. Equation (4) is equivalent to setting

$$\hat{f}\left(X,\left\{X_i\left(\{X_{i0}\},t\right)\right\}\right) = \frac{1}{n}\sum_i \delta(\mathbf{v}-\mathbf{v}_{i0})\ \delta(\mathbf{x}-\mathbf{x}_{i0}-\mathbf{v}_{i0}\ t) + \text{nonsingular term} . \qquad \text{(A-3)}$$

Although this function is often used in the literature[7], this function reduces (A-1'), (A-2') etc. to $f(X, t=0) + \cdots$, $\Delta(1,2)\ f(X_1,\ t=0) +$ nonsingular terms, etc.

Thus we may claim that the singularity of $\hat{f}$ or $\tilde{P}$ can not be expressed in any simple way with functions of initial coordinates $\{X_{i0}\}$ and time.

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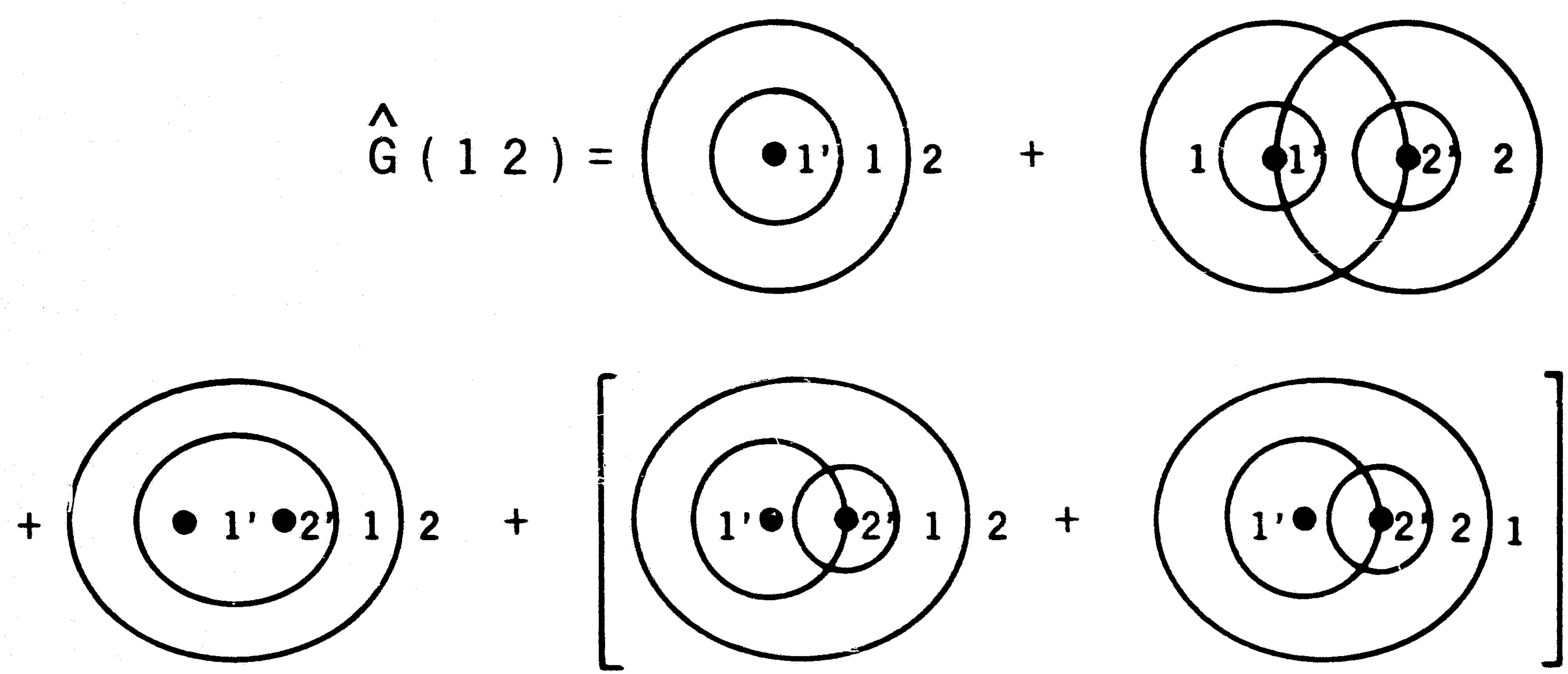

Fig I

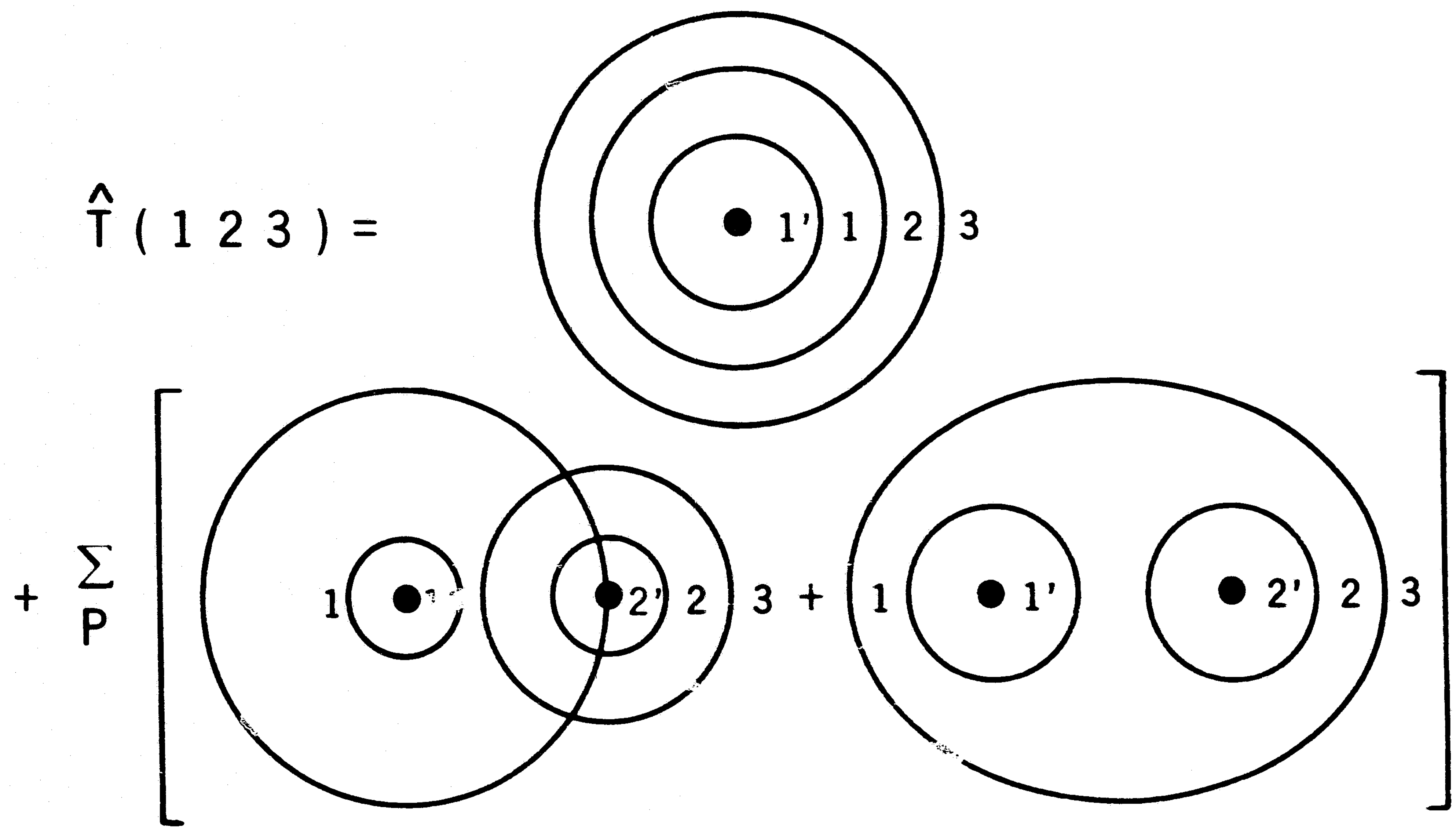

Fig. II

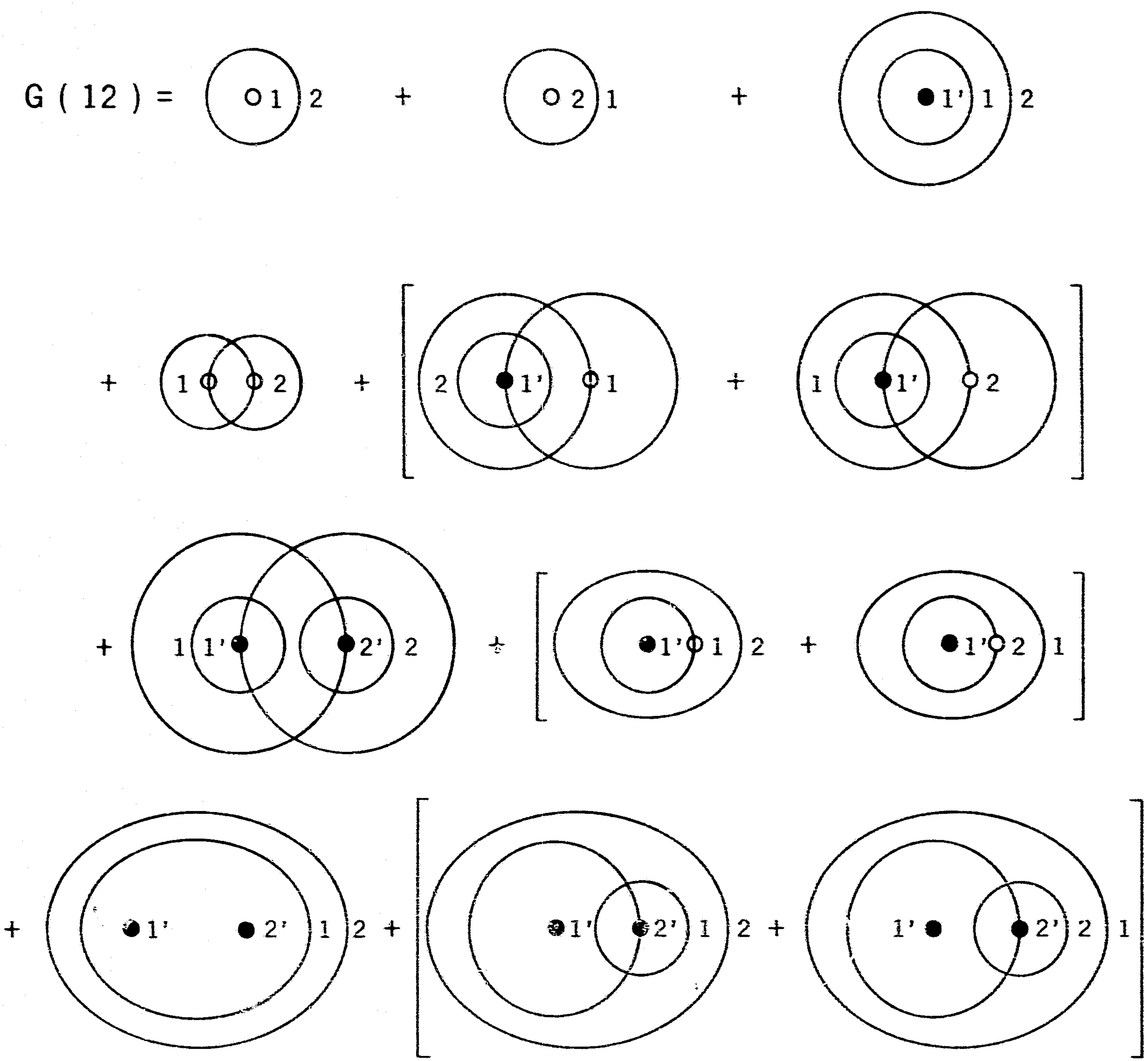

Fig. III

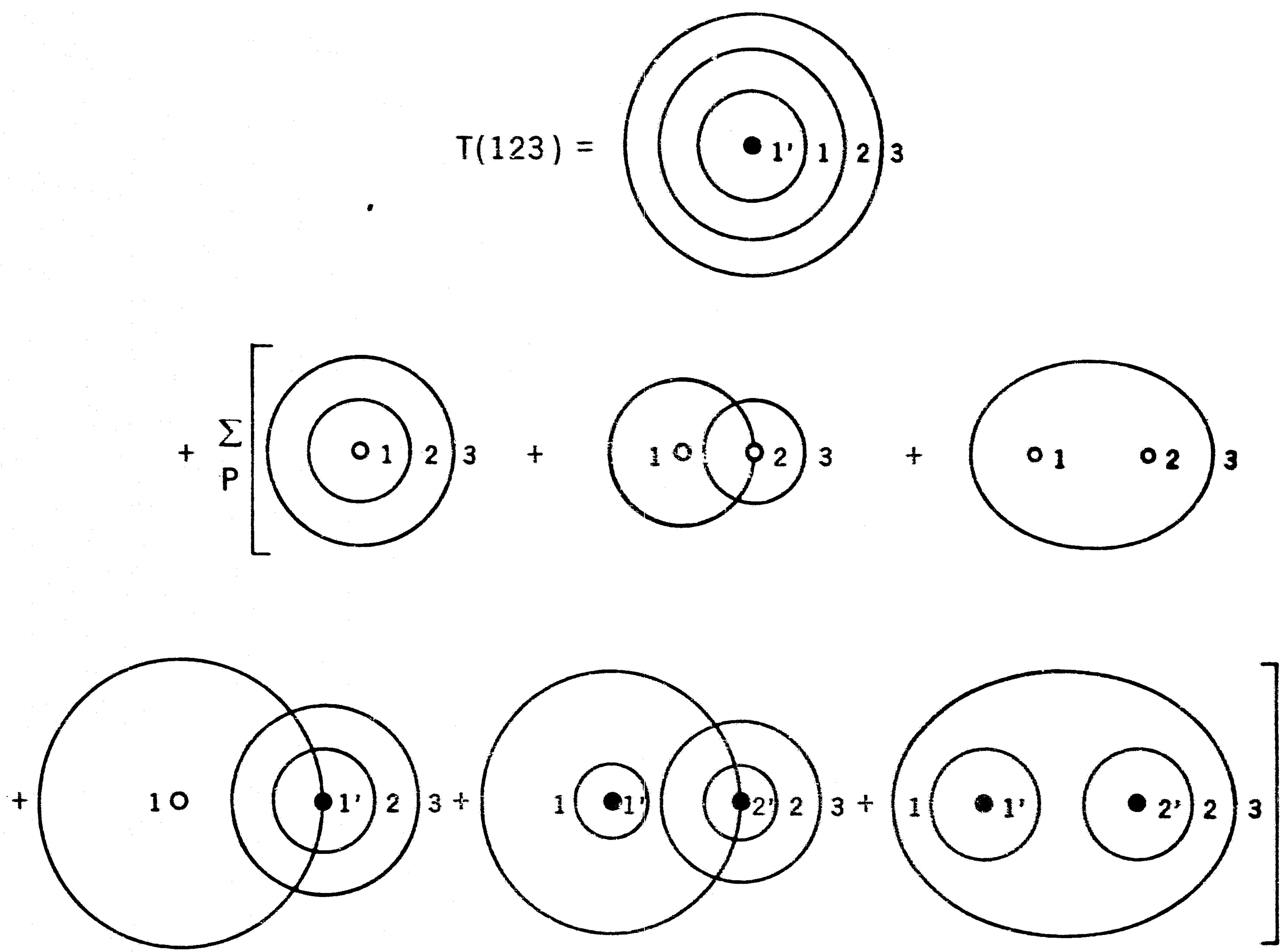

Fig IV